Jesse Benitez Electricity and Magnetism

AP Review Sheet

Chapter 10: DC Circuits

Background/Summary: In this unit, you will use what you have learned about individual electrical components (batteries, resistors and capacitors) to analyze DC Circuits.

Quick Refresher on Electrical Components:

Batteries: Create E fields that motivate charge to move within circuit
Resistors: Limit current and dissipate energy
Capacitors: Store electrical energy in electric field

Key Concepts:

1. **Current** = the amount of charge that passes through a point per unit time.

$$i = \Delta q/\Delta t = coulombs/second = amps$$

* When analyzing circuits using the framework of conventional current, current is defined as the motion of positive charges. Thus, current flows from high voltage to low voltage.

2. Parallel and Series Components:

Parallel Components: share voltage differences
Series Components: share current (and Q for capacitors)

3. Alternating Current **(AC)** vs. Direct Current **(DC)**

AC: Electric field alters in direction
DC: Electric field is always in the same direction

*For the AP test, circuits are always DC.

4. **Branches** and **Nodes**

Branch: Section where current is the same everywhere
Node: Junction where current can be split up or added to

5. **Kirchoff's Laws**

Kirchoff's Laws are used to analyze complex circuits using a combination of loop and node equations.

They comprise of the following 2 equations:

$$\sum i(into \cdot node) = \sum i(out \cdot of \cdot node)$$

$$\sum V(around \cdot closed \cdot loop) = 0$$

Approach for Kirchoff's Laws:

- Step 0: Remove all meters
- Step 1: Define a current for each branch
- Step 2: Write out node equations for each node*
- Step 3: Identify and label loops you will use
- Step 4: Use arrow to denote which direction you are traversing around loops**
- Step 5: Solve using matrices

*There will always be fewer independent node equations than there are nodes
**If there is a battery in the loop, start at the low voltage end and go through the battery when possible. This will ensure that the EMF is positive.

6. Using **Matrices** to Solve Kirchoff's Law Problems

When Kirchoff's Laws are viewed in conjunction for a particular problem, they form systems of equations. Matrices are a helpful tool for solving systems of equations, one that can help you avoid the trials and tribulations of brute force algebra.

To solve Kirchoff's Law problems using matrices, do the following:

--Begin by rewriting each equation so their i_o term is in the first column, its i_2 term is in the second column, etc., and its voltage term (if there is one) is on the right side of the equal sign.

Our equations become:

$$\varepsilon - R_1 i_o - R_2 i_2 = 0 \quad \text{becomes} \quad R_1 i_o + R_2 i_2 + 0 i_3 = \varepsilon$$

$$R_2 i_2 - R_3 i_3 - R_4 i_3 = 0 \quad \text{becomes} \quad 0 i_o + R_2 i_2 - (R_3 + R_4) i_3 = 0$$

$$i_o = i_2 + i_3 \quad \text{becomes} \quad i_o - i_2 - i_3 = 0$$

--Put the information into a matrix:

$$\begin{array}{ccc} i_o \text{ column} & i_2 \text{ column} & i_3 \text{ column} \end{array} \qquad \qquad \text{voltage column}$$

$$\begin{vmatrix} R_1 & R_2 & 0 \\ 0 & R_2 & -(R_3 + R_4) \\ 1 & -1 & -1 \end{vmatrix} \begin{vmatrix} i_o \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} \varepsilon \\ 0 \\ 0 \end{vmatrix}$$

Once you have the matrix set up, you have two options:

1. Manual Evaluation of the Matrices
2. Manipulation of the Matrix Using a Calculator

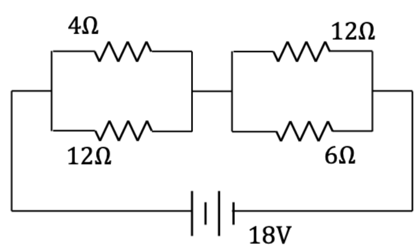
*For review on these topics, go to slides 23-27 on Fletch's Chapter 28 slide show

Important Equations:

Definition of Capacitance:	$\Delta V = Q/C$
Capacitors in Parallel:	$C_p = \sum C_i$
Capacitors in Series:	$1/C_S = \sum 1/C_i$
Definition of Current:	$i = dQ/dt$
Potential Energy in a Capacitor:	$U_C = 1/2(Q\Delta V)) = 1/2(C(\Delta V)^2)$
Resistance in a Wire:	$R = (pl)/A$
Ohm's Law:	$i = \Delta V/R$
Resistors in Series:	$R_S = \sum R_i$
Resistors in Parallel:	$1/R_p = \sum 1/R_i$
Electrical Power:	$P = i\Delta V$

Practice Problems:

1.



The equivalent resistance of the four resistors, connected as shown above, is:

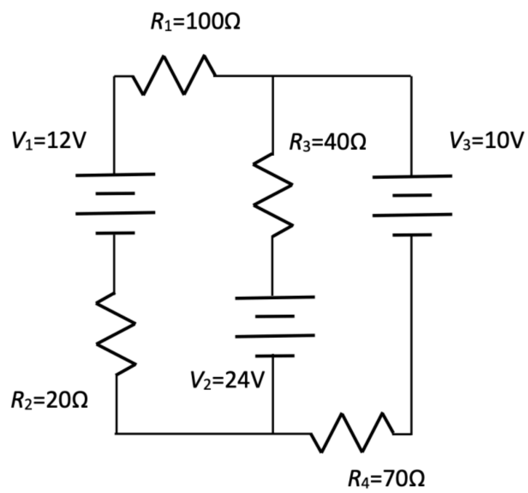
a. 7 Ω
b. 4 Ω
c. 2.7 Ω
d. 8.5 Ω
e. 14.4 Ω

2.

Two parallel plate capacitors, C_1 and C_2, with different capacitances, are connected in series to a battery and allowed to charge fully. If C_2 has twice the capacitance of C_1, which of the following statements must be true?

a. The potential across the plates of each capacitor must be equal.
b. The charge located on the plates of C_2 must be twice the charge located on the plates of C_1.
c. The electric potential across C_2 must be twice the potential across C_1.
d. The electric field strength across C_2 must be twice that of the field across C_1.
e. The energy stored in C_2 must be half that of the energy stored in C_1.

3.



Examine the complex circuit shown here.

a. Use a Kirchhoff's analysis to write, *but do not solve*, a series of non-redundant equations that would allow you to solve for the currents in each branch of the circuit.

Solutions to Practice Problems:

1. The correct answer is *a*. Begin by solving for the equivalent resistances for the parallel combinations of resistors.

$$\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{R_{equivalent}}$$

$$\frac{1}{4}+\frac{1}{12}=\frac{1}{R_{equivalent}} \rightarrow R_{equivalent}=3\Omega$$

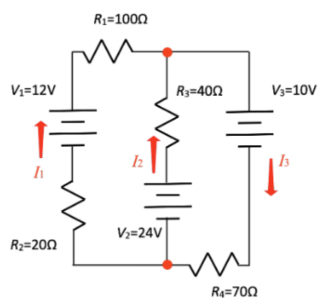
$$\frac{1}{12}+\frac{1}{6}=\frac{1}{R_{equivalent}} \rightarrow R_{equivalent}=4\Omega$$

Use these equivalent resistances (now functioning as two resistors in series) to determine overall resistance.

$$R_1 + R_2 = R_{equivalent}$$
$$R_{equivalent} = 3\Omega + 4\Omega = 7\Omega$$

2. The correct answer is *e*. For capacitors in series, the current, and thus the charge on the plates of each capacitor, are equal. Using this to analyze each answer:
   a. according to $Q=VC$, C_2 has a smaller potential across its plates.
   b. the charge on C_2 is *not* twice that of C_1.
   c. again, according to $Q=VC$, C_2 has a smaller potential across its plates, not a larger one.
   d. the electric field of C_2 is not necessarily twice that of C_1. E depends both potential V and plate separation d, about which we don't know in this problem.
   e. the energy in C_2 *is* half the energy stored in C_1, according to $U = \frac{1}{2}\frac{Q^2}{C}$. With twice the capacitance in C_2 and equal charge Q, the energy stored in C_2 is necessarily half that of C_1.

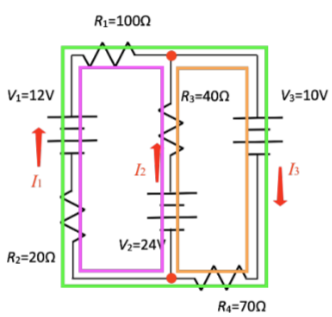
3. The set up is the only important part of this question (the rest is just calculator work or matrix math). To solve complex circuits, we can use Kirchoff's Laws. For this particular circuit, we can use the approach described above. First, identify the branches (where the current is equal) and draw an arrow for direction you choose. An example is shown here:



From this, we can determine:

$$I_1 + I_2 = I_3$$

Having gained all we can from the node equation, we can now turn to the Loop equation. The loops in this circuit are colored below.



Because there are 3 currents, we will need 2 more equations (we will use pink and orange).

$$+12V - 100I_1 + 40I_2 - 24V - 20I_1 = 0$$
$$+24V - 40I_2 - 10V - 70I_3 = 0$$

Using these 3 equations, we would be able to solve for all 3 currents.